Convergence analysis of Tikhonov regularization for nonlinear statistical inverse learning problems

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Abstract

We consider a nonlinear statistical inverse learning problem, where we observe the noisy image of a function $f$ through a nonlinear operator $A$ at some random design points. We discuss the widely used Tikhonov regularization or Method of Regularization approach to reconstruct the estimator of the function $f$ for the nonlinear ill-posed inverse problem. The estimator is defined as the minimizer of a Tikhonov functional, which is the sum of a nonlinear data misfit term and a quadratic penalty term. We develop a theoretical analysis for the minimizer of the nonlinear Tikhonov regularization scheme using the Ansatz of reproducing kernel Hilbert spaces. We discuss the optimal convergence rates for the proposed scheme over the regularity classes defined through appropriate source conditions in the vector-valued function setting.