Conditional Negative Definiteness in Multivariate Geostatistics revisited: the Pseudo Cross-Variogram

Christopher Dörr
University of Mannheim, Mannheim, Germany

Pseudo cross-variograms are a useful tool for the analysis of multivariate random fields, and appear naturally in spatial extreme value theory. We give a necessary and sufficient criterion for a matrix-valued function to be a pseudo cross-variogram, and further provide a Schoenberg-type result connecting pseudo cross-variograms and multivariate correlation functions. This connection adds another dimension to the range of applications of pseudo cross-variograms, that is, the construction of covariance models for multivariate random fields. We illustrate this point via several examples. In particular, we present multivariate extensions of Gneiting's popular space-time covariance model.

The underlying key ingredient for the above results is a stronger notion of conditional negative definiteness for matrix-valued functions than the one for cross-variograms, which have been predominantly used in geostatistical literature. We discuss this stronger notion, and provide a general construction principle for conditionally negative definite matrix-valued kernels, which we use to reinterpret previous covariance modeling proposals.